



Mathematics guidance: key stages 1 and 2

Non-statutory guidance for the national curriculum in England

Year 3

June 2020

What is included in this document?

This document is one chapter of the full publication Mathematics guidance: key stages 1 and 2 Non-statutory guidance for the national curriculum in England.

An overview of the ready-to-progress criteria for all year groups is provided below, followed by the specific guidance for year 3.

To find out more about how to use this document, please read the introductory chapter.

Ready-to-progress criteria: year 1 to year 6

The table below is a summary of the ready-to-progress criteria for all year groups.

Strand	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
NPV	1NPV–1 Count within 100, forwards and backwards, starting with any number.		3NPV-1 Know that 10 tens are equivalent to 1 hundred, and that 100 is 10 times the size of 10; apply this to identify and work out how many 10s there are in other three- digit multiples of 10.	4NPV-1 Know that 10 hundreds are equivalent to 1 thousand, and that 1,000 is 10 times the size of 100; apply this to identify and work out how many 100s there are in other four-digit multiples of 100.	5NPV-1 Know that 10 tenths are equivalent to 1 one, and that 1 is 10 times the size of 0.1. Know that 100 hundredths are equivalent to 1 one, and that 1 is 100 times the size of 0.01. Know that 10 hundredths are equivalent to 1 tenth, and that 0.1 is 10 times the size of 0.01.	6NPV-1 Understand the relationship between powers of 10 from 1 hundredth to 10 million, and use this to make a given number 10, 100, 1,000, 1 tenth, 1 hundredth or 1 thousandth times the size (multiply and divide by 10, 100 and 1,000).
		2NPV-1 Recognise the place value of each digit in two-digit numbers, and compose and decompose two-digit numbers using standard and non-standard partitioning. →	3NPV-2 Recognise the place value of each digit in <i>three</i> -digit numbers, and compose and decompose <i>three</i> -digit numbers using standard and non-standard partitioning. →	4NPV-2 Recognise the place value of each digit in <i>four</i> -digit numbers, and compose and decompose <i>four</i> -digit numbers using standard and non-standard partitioning. →	5NPV-2 Recognise the place value of each digit in numbers with up to 2 decimal places, and compose and decompose numbers with up to 2 decimal places using standard and non-standard partitioning. →	6NPV–2 Recognise the place value of each digit in numbers up to 10 million, including decimal fractions, and compose and decompose numbers up to 10 million using standard and non-standard partitioning.
	1NPV-2 Reason about the location of numbers to 20 within the linear number system, including comparing using < > and =	2NPV-2 Reason about the location of any two- digit number in the linear number system, including identifying the previous and next multiple of 10.	3NPV-3 Reason about the location of any <i>three</i> - digit number in the linear number system, including identifying the previous and next multiple of 100 and 10.	4NPV-3 Reason about the location of any <i>four</i> - digit number in the linear number system, including identifying the previous and next multiple of 1,000 and 100, and rounding to the nearest of each.	5NPV–3 Reason about the location of any number with up to 2 decimals places in the linear number system, including identifying the previous and next multiple of 1 and 0.1 and rounding to the nearest of each.	6NPV–3 Reason about the location of any number up to 10 million, including decimal fractions, in the linear number system, and round numbers, as appropriate, including in contexts.

Strand	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
NPV			3NPV-4 Divide 100 into 2, 4, 5 and 10 equal parts, and read scales/number lines marked in multiples of 100 with 2, 4, 5 and 10 equal parts.	4NPV-4 Divide 1,000 into 2, 4, 5 and 10 equal parts, and read scales/number lines marked in multiples of 1,000 with 2, 4, 5 and 10 equal parts.	5NPV-4 Divide 1 into 2, 4, 5 and 10 equal parts, and read scales/number lines marked in units of 1 with 2, 4, 5 and 10 equal parts.	6NPV–4 Divide powers of 10, from 1 hundredth to 10 million, into 2, 4, 5 and 10 equal parts, and read scales/number lines with labelled intervals divided into 2, 4, 5 and 10 equal parts.
					5NPV–5 Convert between units of measure, including using common decimals and fractions.	
NF	1NF–1 Develop fluency in addition and subtraction facts within 10. →	2NF–1 Secure fluency in addition and subtraction facts within 10, through continued practice. →	<u>3NF-1</u> Secure fluency in addition and subtraction facts that bridge 10, through continued practice.			
	1NF-2 Count forwards and backwards in multiples of 2, 5 and 10, up to 10 multiples, beginning with any multiple, and count forwards and backwards through the odd numbers.		3NF-2 Recall multiplication facts, and corresponding division facts, in the 10, 5, 2, 4 and 8 multiplication tables, and recognise products in these multiplication tables as multiples of the corresponding number.	4NF-1 Recall multiplication and division facts up to 12 × 12, and recognise products in multiplication tables as multiples of the corresponding number.	5NF–1 Secure fluency in multiplication table facts, and corresponding division facts, through continued practice.	
				4NF-2 Solve division problems, with two-digit dividends and one-digit divisors, that involve remainders, and interpret remainders appropriately according to the context.		
			3NF-3 Apply place-value knowledge to known additive and multiplicative number facts (scaling facts by 10). →	4NF-3 Apply place-value knowledge to known additive and multiplicative number facts (scaling facts by 100) →	5NF–2 Apply place-value knowledge to known additive and multiplicative number facts (scaling facts by 1 tenth or 1 hundredth).	

Strand	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
AS	1AS–1 Compose numbers to 10 from 2 parts, and partition numbers to 10 into parts, including recognising odd and even numbers.	2AS–1 Add and subtract across 10.	<u>3AS–1</u> Calculate complements to 100.			6AS/MD-1 Understand that 2 numbers can be related additively or multiplicatively, and quantify additive and multiplicative relationships (multiplicative relationships restricted to multiplication by a whole number).
	1AS-2 Read, write and interpret equations containing addition (+), subtraction (-) and equals (=) symbols, and relate additive expressions and equations to real-life contexts.	2AS–2 Recognise the subtraction structure of 'difference' and answer questions of the form, "How many more?".	<u>3AS-2</u> Add and subtract up to three-digit numbers using columnar methods.			6AS/MD–2 Use a given additive or multiplicative calculation to derive or complete a related calculation, using arithmetic properties, inverse relationships, and place-value understanding.
		2AS–3 Add and subtract within 100 by applying related one-digit addition and subtraction facts: add and subtract only ones or only tens to/from a two- digit number.	3AS–3 Manipulate the additive relationship: Understand the inverse relationship between addition and subtraction, and how both relate to the part–part–whole structure. Understand and use the commutative property of addition, and understand the related property for subtraction.			6AS/MD–3 Solve problems involving ratio relationships.
		2AS-4 Add and subtract within 100 by applying related one-digit addition and subtraction facts: add and subtract any 2 two- digit numbers.				6AS/MD-4 Solve problems with 2 unknowns.

Strand	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
MD		2MD–1 Recognise repeated addition contexts, representing them with multiplication equations and calculating the product, within the 2, 5 and 10 multiplication tables.	<u>3MD-1</u> Apply known multiplication and division facts to solve contextual problems with different structures, including quotitive and partitive division.	4MD-1 Multiply and divide whole numbers by 10 and 100 (keeping to whole number quotients); understand this as equivalent to making a number 10 or 100 times the size. →	5MD–1 Multiply and divide numbers by 10 and 100; understand this as equivalent to making a number 10 or 100 times the size, or 1 tenth or 1 hundredth times the size.	For year 6, MD ready-to- progress criteria are combined with AS ready- to-progress criteria (please see above).
		2MD–2 Relate grouping problems where the number of groups is unknown to multiplication equations with a missing factor, and to division equations (quotitive division).		4MD–2 Manipulate multiplication and division equations, and understand and apply the commutative property of multiplication.	5MD–2 Find factors and multiples of positive whole numbers, including common factors and common multiples, and express a given number as a product of 2 or 3 factors.	
				4MD–3 Understand and apply the distributive property of multiplication. →	5MD–3 Multiply any whole number with up to 4 digits by any one-digit number using a formal written method.	
					5MD–4 Divide a number with up to 4 digits by a one-digit number using a formal written method, and interpret remainders appropriately for the context.	

Strand	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
F			<u>3F-1</u> Interpret and write proper fractions to represent 1 or several parts of a whole that is divided into equal parts.			6F–1 Recognise when fractions can be simplified, and use common factors to simplify fractions.
			3F-2 Find unit fractions of quantities using known division facts (multiplication tables fluency). →		5F–1 Find non-unit fractions of quantities.	6F–2 Express fractions in a common denomination and use this to compare fractions that are similar in value.
			3F-3 Reason about the location of any fraction within 1 in the linear number system. →	4F–1 Reason about the location of mixed numbers in the linear number system.		6F–3 Compare fractions with different denominators, including fractions greater than 1, using reasoning, and choose between reasoning and common denomination as a comparison strategy.
				4F–2 Convert mixed numbers to improper fractions and vice versa.	5F–2 Find equivalent fractions and understand that they have the same value and the same position in the linear number system.	
			<u>3F-4</u> Add and subtract fractions with the same denominator, within 1. →	4F–3 Add and subtract improper and mixed fractions with the same denominator, including bridging whole numbers.	5F–3 Recall decimal fraction equivalents for $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{1}{10}$, and for multiples of these proper fractions.	
G	1G–1 Recognise common 2D and 3D shapes presented in different orientations, and know that rectangles, triangles, cuboids and pyramids are not always similar to one another. →	2G-1 Use precise language to describe the properties of 2D and 3D shapes, and compare shapes by reasoning about similarities and differences in properties.	<u>3G-1</u> Recognise right angles as a property of shape or a description of a turn, and identify right angles in 2D shapes presented in different orientations.		5G–1 Compare angles, estimate and measure angles in degrees (°) and draw angles of a given size.	

Strand	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
G					5G–2 Compare areas and calculate the area of rectangles (including squares) using standard units.	
	1G–2 Compose 2D and 3D shapes from smaller shapes to match an example, including manipulating shapes to place them in particular orientations. →		3G-2 Draw polygons by joining marked points, and identify parallel and perpendicular sides. →	4G–1 Draw polygons, specified by coordinates in the first quadrant, and translate within the first quadrant. →		6G–1 Draw, compose, and decompose shapes according to given properties, including dimensions, angles and area, and solve related problems.
				4G–2 Identify regular polygons, including equilateral triangles and squares, as those in which the side-lengths are equal and the angles are equal. Find the perimeter of regular and irregular polygons.		
				4G–3 Identify line symmetry in 2D shapes presented in different orientations. Reflect shapes in a line of symmetry and complete a symmetric figure or pattern with respect to a specified line of symmetry.		

Year 3 guidance

Ready-to-progress criteria

Year 2 conceptual prerequisite	Year 3 ready-to-progress criteria	Future applications
Know that 10 ones are equivalent to 1 ten, and that 40 (for example) can be composed from 40 ones or 4 tens. Know how many tens there are in multiples of 10 up to 100.	<u>3NPV-1</u> Know that 10 tens are equivalent to 1 hundred, and that 100 is 10 times the size of 10; apply this to identify and work out how many 10s there are in other three-digit multiples of 10.	Solve multiplication problems that that involve a scaling structure, such as 'ten times as long'.
Recognise the place value of each digit in <i>two</i> -digit numbers, and compose and decompose <i>two</i> -digit numbers using standard and non-standard partitioning.	<u>3NPV-2</u> Recognise the place value of each digit in <i>three</i> -digit numbers, and compose and decompose <i>three</i> -digit numbers using standard and non-standard partitioning.	Compare and order numbers. Add and subtract using mental and formal written methods.
Reason about the location of any <i>two</i> -digit number in the linear number system, including identifying the previous and next multiple of 10.	<u>3NPV–3</u> Reason about the location of any <i>three</i> -digit number in the linear number system, including identifying the previous and next multiple of 100 and 10.	Compare and order numbers. Estimate and approximate to the nearest multiple of 1,000, 100 or 10.
Count in multiples of 2, 5 and 10.	<u>3NPV-4</u> Divide 100 into 2, 4, 5 and 10 equal parts, and read scales/number lines marked in multiples of 100 with 2, 4, 5 and 10 equal parts.	Read scales on graphs and measuring instruments.

Year 2 conceptual prerequisite	Year 3 ready-to-progress criteria	Future applications
Add and subtract across 10, for example: 8+5=13 13-5=8	<u>3NF–1</u> Secure fluency in addition and subtraction facts that bridge 10, through continued practice.	Add and subtract mentally where digits sum to more than 10, for example: 26+37=63
		Add and subtract across other powers of 10, without written methods, for example: 1.3 - 0.4 = 0.9 Add within a column during columnar addition when the column sums to more than 10 (regrouping), for example, for: 126 + 148
		Subtract within a column during columnar subtraction when the minuend of the column is smaller than the subtrahend (exchanging), for example, for: 45 3 – 12 4
Calculate products within the 2, 5 and 10 multiplication tables.	3NF-2 Recall multiplication facts, and corresponding division facts, in the 10, 5, 2, 4 and 8 multiplication tables, and recognise products in these multiplication tables as multiples of the corresponding number.	Use multiplication facts during application of formal written layout. Use division facts during short division and long division.
Automatically recall addition and subtraction facts within 10, and across 10. Unitise in tens: understand that 10 can be thought of as a single unit of 1 ten.	<u>3NF-3</u> Apply place-value knowledge to known additive and multiplicative number facts (scaling facts by 10), for example: 80 + 60 = 140 140 - 60 = 80 $30 \times 4 = 120$ $120 \div 4 = 30$	Apply place-value knowledge to known additive and multiplicative number facts (scaling facts by 100), for example: 8 + 6 = 14 and $14 - 6 = 8so800 + 600 = 1,4001,400 - 600 = 8003 \times 4 = 12 and 12 \div 4 = 3$
		so 300 × 4 = 1,200 1,200 ÷ 4 = 300

Year 2 conceptual prerequisite	Year 3 ready-to-progress criteria	Future applications
Automatically recall number bonds to 9 and to 10. Know that 10 ones are equivalent to 1 ten, and 10 tens are equivalent to 1 hundred.	3AS-1 Calculate complements to 100, for example: 46+? = 100	Calculate complements to other numbers, particularly powers of 10. Calculate how much change is due when paying for an item.
Automatically recall addition and subtraction facts within 10 and across 10. Recognise the place value of each digit in two- and three-digit numbers. Know that 10 ones are equivalent to 1 ten, and 10 tens are equivalent to 1 hundred.	3AS–2 Add and subtract up to three-digit numbers using columnar methods.	Add and subtract other numbers, including four- digits and above, and decimals, using columnar methods.
Have experience with the commutative property of addition, for example, have recognised that $3+2$ and 2+3 have the same sum. Be able to write an equation in different ways, for example, 2+3=5 and $5=2+3Write equations torepresent addition andsubtraction contexts.$	3AS-3 Manipulate the additive relationship: Understand the inverse relationship between addition and subtraction, and how both relate to the part-part-whole structure. Understand and use the commutative property of addition, and understand the related property for subtraction.	All future additive reasoning.
Recognise repeated addition contexts and represent them with multiplication equations. Relate grouping problems where the number of groups is unknown to multiplication equations with a missing factor, and to division equations (quotitive division).	<u>3MD–1</u> Apply known multiplication and division facts to solve contextual problems with different structures, including quotitive and partitive division.	

Year 2 conceptual prerequisite	Year 3 ready-to-progress criteria	Future applications
	3F–1 Interpret and write proper fractions to represent 1 or several parts of a whole that is divided into equal parts.	Use unit fractions as the basis to understand non- unit fractions, improper fractions and mixed numbers, for example: $\frac{2}{5}$ is 2 one-fifths $\frac{6}{5}$ is 6 one-fifths, so $\frac{6}{5} = 1\frac{1}{5}$
	<u>3F–2</u> Find unit fractions of quantities using known division facts (multiplication tables fluency).	Apply knowledge of unit fractions to non-unit fractions.
Reason about the location of whole numbers in the linear number system.	3F–3 Reason about the location of any fraction within 1 in the linear number system.	Compare and order fractions.
Automatically recall addition and subtraction facts within 10. Unitise in tens: understand that 10 can be thought of as a single unit of 1 ten, and that these units can be added and subtracted.	<u>3F-4</u> Add and subtract fractions with the same denominator, within 1.	Add and subtract improper and mixed fractions with the same denominator, including bridging whole numbers.
Recognise standard and non-standard examples of 2D shapes presented in different orientations. Identify similar shapes.	<u>3G-1</u> Recognise right angles as a property of shape or a description of a turn, and identify right angles in 2D shapes presented in different orientations.	Compare angles. Estimate and measure angles in degrees.
Compose 2D shapes from smaller shapes to match an exemplar, rotating and turning over shapes to place them in specific orientations.	<u>3G–2</u> Draw polygons by joining marked points, and identify parallel and perpendicular sides.	Find the area or volume of a compound shape by decomposing into constituent shapes. Find the perimeter of regular and irregular polygons.

3NPV-1 Equivalence of 10 hundreds and 1 thousand

Know that 10 tens are equivalent to 1 hundred, and that 100 is 10 times the size of 10; apply this to identify and work out how many 10s there are in other three-digit multiples of 10.

3NPV–1 Teaching guidance

Pupils need to experience:

- what 100 items looks like
- making a unit of 1 hundred out of 10 units of 10, for example using 10 bundles of 10 straws to make 100, or using ten 10-value place-value counters



Figure 1: ten 10-value place-value counters in a tens frame

Language focus

"10 tens is equal to 1 hundred."

Pupils must then be able to work out how many tens there are in other three-digit multiples of 10.



Figure 2: eighteen 10-value place-value counters in 2 tens frames



The reasoning here can be described as grouping or repeated addition – pupils group or add 10 tens to make 100, then add another group of 8 tens.

Pupils need to be able to apply this reasoning to measures contexts, as shown in the <u>**3NPV-1**</u> below. It is important for pupils to understand that there are tens within this new unit of 100, in different contexts.

Pupils should be able to explain that numbers such as 180 and 300 are multiples of 10, because they are each equal to a whole number of tens. They should be able to identify multiples of 10 based on the fact that they have a zero in the ones place.

As well as understanding 100 and other three-digit multiples of 10 in terms of grouping and repeated addition, pupils should begin to describe 100 as 10 times the size of 10.



Figure 3: place-value chart illustrating the scaling relationship between ones, tens and hundreds

Language focus

"100 is 10 times the size of 10."

Making connections

Learning to identify the number of tens in three-digit multiples of 10 should be connected to pupils understanding of multiplication and the grouping structure of division (**2MD-1**). Pupils should, for example, be able to represent 180 as 18 tens using the multiplication equations $180 = 18 \times 10$ or $180 = 10 \times 18$, and be able to write the corresponding division equations $180 \div 10 = 18$. In <u>3MD-1</u> they will learn that $180 \div 10 = 18$ can represent the structure of 180 divided into groups of 10 (quotitive division), as here, and that it can also represent 180 shared into 10 equal shares of 18 each (partitive division).

3NPV-1 Example assessment questions

- 1. How many 10cm lengths can a 310cm length of ribbon be cut into?
- 2. The school office sells 52 poppies for 10p each. How much money have they collected altogether?
- 3. I take 10ml of medicine every day. How many days will a 250ml bottle last?
- 4. Marek is 2 years old, and has a mass of 10kg. His father's mass is 10 times as much. What is the mass of Marek's father?
- 5. Janey saves up £100. This is 10 times as much money as her brother has. How much money does her brother have?
- 6. Circle the numbers that are multiples of 10. Explain your answer.

640 300 105 510 330 409 100 864

3NPV-2 Place value in three-digit numbers

Recognise the place value of each digit in *three*-digit numbers, and compose and decompose *three*-digit numbers using standard and non-standard partitioning.

3NPV-2 Teaching guidance

Pupils should be able to identify the place value of each digit in a three-digit number. They must be able to combine units of ones, tens and hundreds to compose three-digit numbers, and partition three-digit numbers into these units. Pupils need to experience variation in the order of presentation of the units, so that they understand that 40 + 300 + 2 is equal to 342, not 432.





Pupils also need to solve problems relating to subtraction of any single place-value part from the whole number, for example:



As well as being able to partition numbers in the 'standard' way (into individual placevalue units), pupils must also be able to partition numbers in 'non-standard' ways, and carry out related addition and subtraction calculations, for example:





Figure 5: partitioning 830 into 430 and 400

Figure 6: partitioning 654 into 620 and 34

You can find out more about fluency and recording for these calculations here in the calculation and fluency section: <u>Number, place value and number facts: 3NPV–2 and 3NF–3</u>

3NPV-2 Example assessment questions

1. What number is represented by these counters?



2. What number is represented by this expression?

 $1\!+\!10\!+\!10\!+\!100\!+\!100\!+\!10\!+\!10$

3. Fill in the missing numbers to complete these partitioning diagrams.



4. Fill in the missing numbers.



100 + 60 + 5 105 + 60

300 + 40 + 2 300 + 24

783-80 783-3

839-9-30 839-39

- 6. There are 365 days in a year. If it rains on 65 days of the year, on how many days does it not rain?
- 7. A bamboo plant was 4m tall. Then it grew by another 83cm. How tall is the bamboo plant now? Express your answer in centimetres.
- 8. In the school library there are 25 books on the trolley and 250 books on the shelves. How many books are there altogether?
- 9. Francesco had 165 marbles. Then he gave 45 marbles to his friend. How many marbles does Francesco have now?
- 10. The tree outside Cecily's house in 308cm tall. How much further would it have to grow to reach the bottom of Cecily's bedroom window, at 3m 68cm?

3NPV–3 Three-digit numbers in the linear number system

Reason about the location of any *three*-digit number in the linear number system, including identifying the previous and next multiple of 100 and 10.

3NPV-3 Teaching guidance

Pupils need to be able to identify or place three-digit numbers on marked number lines with a variety of scales. Pupils should also be able to estimate the value or position of three-digit numbers on unmarked numbers lines, using appropriate proportional reasoning. Pupils should apply this skill to taking approximate readings of scales in measures and statistics contexts, as shown in the <u>3NPV-3</u> below. For more detail on identifying, placing and estimating positions of numbers on number lines, see year 2, **2NPV-2**.

Pupils must also be able to identify which pair of multiples of 100 or 10 a given three-digit number is between. To begin with, pupils can use a number line for support. In this example, for the number 681, pupils must identify the previous and next multiples of 100 and 10.



Figure 7: using a number line to identify the next and previous multiple of 100



Figure 8: using a number line to identify the next and previous multiple of 10



Pupils need to be able to identify previous and next multiples of 100 or 10 without the support of a number line.



Figure 9: identifying the nearest multiple of 100

Finally, pupils should also be able to count forwards and backwards from any three-digit number in steps of 1 or 10. Pay particular attention to counting over 'boundaries', for example:

- 210, 200, 190
- 385, 395, 405

Making connections

Here, pupils must apply their knowledge that 10 tens is equal to 1 hundred (see <u>3NPV-1</u>) to understand that each interval of 100 on a number line or scale is made up of 10 intervals of 10. This also links to <u>3NPV-4</u>, in which pupils need to be able to read scales divided into 2, 4, 5 and 10 equal parts.

3NPV-3 Example assessment questions

1. Fill in the missing numbers.



2. Estimate to fill in the missing numbers.



3. Estimate and mark the position of these numbers on the number line.

 600
 200
 480
 840
 762
 195



4. Fill in the missing numbers.



5. Look at lines A, B and C. Can you estimate how long they are by comparing them to the 1,000cm lines?



6. Estimate the mass, in grams, shown on this weighing scale.



3NPV-4 Reading scales with 2, 4, 5 or 10 intervals

Divide 100 into 2, 4, 5 and 10 equal parts, and read scales/number lines marked in multiples of 100 with 2, 4, 5 and 10 equal parts.

3NPV-4 Teaching guidance

By the end of year 3, pupils must be able to divide 100 into 2, 4, 5 or 10 equal parts. This is important because these are the intervals commonly found on measuring instruments and graph scales.



Figure 10: Bar models showing 100 partitioned into 2, 4, 5 and 10 equal parts

Pupils should practise counting in multiples of 10, 20, 25, and 50 from 0, or from any multiple of these numbers, both forwards and backwards. This is an important step in becoming fluent with these number patterns. Pupils will have been practising counting in multiples of 1, 2 and 5 since year 1, and this supports counting in units of 10, 20 and 50. However, counting in units of 25 is not based on any multiples with which pupils are already familiar, so they typically find this the most challenging.

Language focus

"Twenty-five, fifty, seventy-five, one hundred" needs to be a fluent spoken language pattern, which pupils can continue over 100.

Pupils should be able to apply this skip counting beyond 100, to solve contextual multiplication and division measures problems, as shown in the <u>3NPV-4</u> below (questions 5 and 7). Pupils should also be able to write and solve multiplication and division equations related to multiples of 10, 20, 25 and 50 up to 100.

Pupils need to be able to solve addition and subtraction problems based on partitioning 100 into multiples of 10, 20 and 50 based on known number bonds to 10. Pupils should also have automatic recall of the fact that 25 and 75 are bonds to 100. They should be able to automatically answer a question such as "I have 1m of ribbon and cut off 25cm. How much is left in centimetres?"

Making connections

Dividing 100 into 10 equal parts is also assessed as part of <u>3NPV-1</u>.

Reading scales builds on number line knowledge from <u>**3NPV-3**</u>. Conversely, experience of working with scales with 2, 4, 5 or 10 divisions in this criterion improves pupils' estimating skills when working with unmarked number lines and scales as described in **3NPV-3**.

3NPV-4 Example assessment questions

1. Fill in the missing numbers.



2. What were Jenny and Asif's scores?



3. Miss Scot weighs herself. How much does she weigh, in kilograms?



4. How many centimetres long is the ribbon?



- 5. How many 25p cupcakes can I buy for £5?
- 6. How many 50cm lengths of wood can I cut from a 3m plank?
- 7. We raise £100 at the school fair and divide the money equally between 5 charities. How much does each charity get?
- 8. Fill in the missing numbers.

9.

100÷4	=	× 20 = 1	00 1	$00 \div 50 = $	2	5+=100	C
Stan count	s from 0 in i	multiples of 2	5. Circle th	e numbers	s he will sa	ay.	
100	25	240	155	400	275	505	350

3NF-1 Fluently add and subtract within and across 10

Secure fluency in addition and subtraction facts that bridge 10, through continued practice.

3NF-1 Teaching guidance

Before pupils begin work on columnar addition and subtraction (<u>3AS-1</u>), it is essential that pupils have automatic recall of addition and subtraction facts within and across 10. These facts are required for calculation within the columns in columnar addition and subtraction. All mental calculation also depend on these facts.

Identifying core number facts: columnar addition	Identifying core number facts: columnar subtraction
$ \begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	$ \begin{array}{r} 6 & 1 \\ 7 & 4 & 9 \\ - & 2 & 8 & 6 \\ 4 & 6 & 3 \end{array} $
Figure 11: columnar addition of 465 and 429	Figure 12: columnar subtraction of 286 from 749
Within-column calculations:	Within-column calculations:
5+9=14 6+2+1=9 4+4=8	9-6=3 7-1=6 14-8=6 6-2=4

Pupils should already have automatic recall of addition and subtraction facts within 10, from year 1 (**1NF-1**). In year 2 (**2AS-1**), pupils learnt strategies for addition and subtraction across 10. However, year 3 pupils are likely to need further practice, and reminders of the strategies, to develop sufficient fluency. Pupils should practise until they achieve automaticity in the mental application of these strategies. Without this practice many pupils are likely to still be reliant on counting on their fingers to solve within-column calculations in columnar addition and subtraction.

The full set of addition calculations that pupils need for columnar addition are shown on the next page. The number of facts to be learnt is reduced when commutativity is applied and pupils recognise that 7+5, for example, is the same as 5+7. Automaticity in subtraction facts should also be developed through the application of the relationship between addition and subtraction, for example, pupils should recognise that if 7+5=12 then 12-5=7.

+	0	1	2	3	4	5	6	7	8	9	10
0	0+0	0+1	0+2	0+3	0 + 4	0+5	0+6	0+7	0+8	0+9	0+10
1	1+0	1+1	1+2	1+3	1+4	1+5	1+6	1+7	1+8	1+9	1+10
2	2+0	2+1	2+2	2+3	2 + 4	2+5	2+6	2+7	2+8	2+9	2+10
3	3+0	3+1	3+2	3+3	3 + 4	3+5	3+6	3+7	3+8	3+9	3+10
4	4+0	4 + 1	4 + 2	4+3	4 + 4	4 + 5	4+6	4 + 7	4+8	4+9	4+10
5	5+0	5+1	5+2	5 + 3	5 + 4	5 + 5	5+6	5+7	5+8	5+9	5+10
6	6+0	6+1	6+2	6+3	6+4	6+5	6+6	6+7	6+8	6+9	6+10
7	7+0	7+1	7+2	7+3	7+4	7+5	7+6	7+7	7 + 8	7 + 9	7+10
8	8+0	8+1	8+2	8+3	8+4	8+5	8+6	8+7	8+8	8+9	8+10
9	9+0	9+1	9+2	9+3	9+4	9+5	9+6	9+7	9+8	9+9	9+10
10	10+0	10+1	10+2	10+3	10+4	10+5	10+6	10+7	10+8	10+9	10+10

Pupils need extensive practice to meet this criterion. You can find out more about fluency for addition and subtraction within and across 10 here in the calculation and fluency section: <u>3NF-1</u>

Making connections

Fluency in these addition and subtraction facts is required for within-column calculation in columnar addition and subtraction (3AS-2).

3NF-1 Example assessment questions

- 1. Mr Kahn drove 8km to get to his friend's house, and then drove another 3km with his friend to get to the gym. How far did Mr Kahn drive?
- 2. There are 12 children. 5 of them can ride a bicycle and the rest cannot. How many of the children cannot ride a bicycle?
- 3. Maja had £17. Then she spent £9. How much money does she have left?
- 4. I have 6 metres of red ribbon and 6 metres of blue ribbon. How many metres of ribbon do I have altogether?

5. Hazeem is growing a sunflower and a bean plant. So far, his sunflower plant is 14cm tall and his bean plant is 8cm tall. How much taller is the sunflower plant than the bean plant?

Assessment guidance: For pupils to have met criterion **3NF-1**, they need to be able to add and subtract within and across 10 without counting forwards or backwards in ones on their fingers, on a number line or in their heads. Pupils need to be able to automatically recall the facts within 10, and be able to mentally apply strategies for calculation across 10, with accuracy and speed. Teachers should assess pupils in small groups – simply providing the correct answers to the example questions above does not demonstrate that a pupil has met the criterion. The full set of addition and subtraction facts which children need to be fluent in is shown in the appendix.

3NF-2 Recall of multiplication tables

Recall multiplication facts, and corresponding division facts, in the 10, 5, 2, 4 and 8 multiplication tables, and recognise products in these multiplication tables as multiples of the corresponding number.

3NF-2 Teaching guidance

The national curriculum requires pupils to recall multiplication table facts up to 12×12 , and this is assessed in the year 4 multiplication tables check. In year 3, the focus should be on learning facts in the 10, 5, 2, 4 and 8 multiplication tables.

Language focus

When pupils commit multiplication table facts to memory, they do so using a verbal sound pattern to associate the 3 relevant numbers, for example, "six fours are twenty-four". It is important to provide opportunities for pupils to verbalise each multiplication fact as part of the process of developing fluency.

While pupils are learning the individual multiplication tables, they should also learn that:

• the factors can be written in either order and the product remains the same (for example, we can write $3 \times 4 = 12$ or $4 \times 3 = 12$ to represent the third fact in the 4 multiplication table)

- the products within each multiplication table are multiples of the corresponding number, and be able to recognise multiples (for example, pupils should recognise that 64 is a multiple of 8, but that 68 is not)
- adjacent multiples in, for example, the 8 multiplication table, have a difference of 8



Figure 13: number line and array showing that adjacent multiples of 8 (32 and 40) have a difference of 8

As pupils develop automatic recall of multiplication facts, they should learn how to use these to derive division facts. They must be able to use these 'known division facts' to solve division calculations, instead of using the skip counting method they learnt in year 2 (**2MD–1**). Pupils should learn to make the connection between multiplication and division facts as they learn each multiplication table, rather than afterwards.

Language focus

"4 times 5 is 20, so 20 divided by 5 is 4."

It is useful for pupils to learn the 10 and 5 multiplication tables one after the other, and then the 2, 4 and 8 multiplication tables one after the other. The connections and patterns will help pupils to develop fluency and understanding.

You can find out more about developing automatic recall of multiplication tables here in the calculation and fluency section: 3NF-2

Making connections

Alongside developing fluency in multiplication tables facts and corresponding division facts, pupils must learn to apply them to solve contextual problems with different multiplication and division structures (<u>3MD-1</u>).

In <u>3F-2</u> pupils use known division facts to find unit fractions of quantities, for example $36 \div 4 = 9$, so $\frac{1}{4}$ of 36 = 9.

3NF-2 Example assessment questions

- 1. A spider has 8 legs. If there are 5 spiders, how many legs are there altogether?
- 2. A book costs £5. How much do 6 books cost?
- 3. 18 socks are put into pairs. How many pairs are there?
- 4. Felicity wants to buy a scooter for £60. If she pays with £10 notes, how many notes does she need?
- 5. Circle the numbers that are multiples of 4.

14	24	40	34	16	32	25

Assessment guidance: The multiplication tables check in year 4 will assess pupils' fluency in all multiplication tables. At this stage, teachers should assess fluency in facts within the 10, 5, 2, 4 and 8 multiplication tables. Once pupils can automatically recall multiplication facts, and have covered criterion <u>3MD–1</u>, they should be able to apply their knowledge to contextual questions like those shown here. Teachers should ensure that pupils answer these questions using automatic recall of the appropriate multiplication facts – for question 1, for example, if a pupil counts up in multiples of 8, or draws 5 spiders and counts the legs in ones, the pupil has not met this criterion.

3NF-3 Scaling number facts by 10

Apply place-value knowledge to known additive and multiplicative number facts (scaling facts by 10), for example:

8 + 6 = 14 and $14 - 6 = 8$	$3 \times 4 = 12$ and $12 \div 4 = 3$
SO	SO
80 + 60 = 140 and $140 - 60 = 80$	$30 \times 4 = 120$ and $120 \div 4 = 30$

3NF-3 Teaching guidance

During year 3, pupils develop automaticity in addition and subtraction facts within 20 (<u>3NF-1</u>), and learn to recall multiplication table facts and related division facts for the 10, 5, 2, 4 and 8 multiplication tables. To be ready to progress to year 4, pupils must also be able to combine these facts with unitising in tens, including:

- scaling known additive facts within 10, for example, 90-60=30
- scaling known additive facts that bridge 10, for example, 80 + 60 = 140
- scaling known multiplication tables facts, for example, $30 \times 4 = 120$
- scaling division facts derived from multiplication tables, for example, $120 \div 4 = 30$

For calculations such as 80 + 60 = 140, pupils can begin by using tens frames and counters as they did for calculation across 10 (**2AS–1**), but now using 10-value counters.



Figure 14: tens frames with 10-value counters showing 80 + 60 = 140

8 + 6 = 14	14 - 6 = 8	14 - 8 = 6	
80 + 60 = 140	140 - 60 = 80	140 - 80 = 60	

You can find out more about fluency and recording for these calculations here in the calculation and fluency section: <u>Number, place value and number facts: 3NPV–2 and 3NF–3</u>

Similarly, pupils can use 10-value counters to understand how a known multiplicative fact, such as $3 \times 5 = 15$, relates to a scaled calculation, such as $3 \times 50 = 150$. Pupils should be able reason in terms of unitising in tens.

10	10	10	10	10
10	10	10	10	10
10	10	10	10	10



$3 \times 5 = 15$	$3 \times 5 = 15$
$3 \times 50 = 150$	$30 \times 5 = 150$

Language focus	Language focus
"3 times 5 is equal to 15."	"3 times 5 is equal to 15."
"3 times 5 tens is equal to 15 tens."	"3 tens times 5 is equal to 15 tens."
"15 tens is equal to 150."	"15 tens is equal to 150."

Multiplication calculations in this criterion should all be related to the 5, 10, 2, 4 and 8 multiplication tables. It is important for pupils to understand all of the calculations in this criterion in terms of working with units of 10.

Making connections

This criterion builds on:

- additive fluency within and across 10 (3NF-1)
- <u>3NF-2</u>, where pupils develop fluency in multiplication and division facts
- <u>3NPV-1</u>, where pupils need to be able to work out how many tens there are in any three-digit multiple of 10

Meeting this criterion also requires pupils to be able to fluently multiply whole numbers by 10 ($\underline{3NF-2}$).

3NF-3 Example assessment questions

- 1. A garden table costs £80 and 2 garden chairs each cost £60. How much do the 2 chairs and the table cost altogether?
- 2. 130 people are expected at a concert. So far 70 people have arrived. How many more people are due to arrive?
- 3. A family ticket for a safari park is £40. 3 families go together. How much do the 3 family tickets cost altogether?
- 4. Fill in the missing numbers.



3AS-1 Calculate complements to 100

Calculate complements to 100, for example: 46 + ? = 100

3AS-1 Teaching guidance

Calculating complements to 100 is an important skill, because it is a prerequisite for calculating how much change is due when paying for an item. When pupils calculate complements (the amount needed to complete a total), a common error is to end up with a total that is too large:

- When calculating complements to 100, pupils typically make an extra 'unit' of 10, making 110 instead of 100.
- When finding change from a whole number of pounds, pupils typically make an extra £1, for example, they incorrectly calculate the change due from £5 for a cost of £3.40 as £2.60

It is important for pupils to spend time specifically learning about calculating complements, including the risk of creating 'extra units'. This should begin in year 3, with calculating complements to 100.

Pupils should compare correct calculations with the corresponding common incorrect calculations for complements to 100. They should be able to discuss the pairs of calculations and understand the source of the error in the incorrect calculations.



A shaded 100 grid can be used to show why there are only 9 full tens in the correct complements to 100. The 10th ten is composed of the ones digits.



Figure 18: a 100 grid shaded in 2 colours to represent 62 and 38 as a complement to 100

Once pupils understand why given complements to 100 are correct or not, they should learn to work in steps to calculate complements themselves:

- 1. First make 10 ones.
- 2. Then work out the number of additional tens needed. Pupils must understand that the tens digits should bond to 9, not to 10.
- 3. Check that the 2 numbers sum to 100.



Figure 19: calculating a complement to 100

Language focus

"First we make 10 ones. The ones digits add up to 1 ten, so we need 9 more tens."

Making connections

Pupils will need to calculate the majority of two-digit complements to 100 as described above. However, pupils should memorise the pair 75 and 25 (see <u>3NPV-4</u>).

3AS–1 Example assessment questions

1. Which of these are correct complements to 100 and which have an extra 10? Tick the correct column. Explain your answers.

	Correct bond to 100	Incorrect bond to 100 (extra 10)	Explanation
28 + 72			
61 + 49			
55 + 45			
43 + 67			
84 + 16			
39 + 71			

2. Fill in the missing numbers.

65+=100	100 – 29 =

100 = 42 +] = 100 - 83
------------	--	--	--	--------------

- 3. A dressmaker had 1m of ribbon. Then she used 22cm of it. How many centimetres of ribbon does she have left?
- 4. A toy shop sells ping-pong balls for 65p each. If I use a £1 coin to pay for a ping-pong ball, how much change will I get, in pence?
- 5. Mr Jones has 100 stickers. 47 of them are gold and the rest are silver. How many are silver?

3AS–2 Columnar addition and subtraction

Add and subtract up to three-digit numbers using columnar methods.

3AS-2 Teaching guidance

Pupils must learn to add and subtract using the formal written methods of columnar addition and columnar subtraction. Pupils should master columnar addition, including calculations involving regrouping (some columns sum to 10 or more), before learning columnar subtraction. However, guidance here is combined due to the similarities between the two algorithms.

Beginning with calculations that do not involve regrouping (no columns sum to 10 or more) or exchange (no columns have a minuend smaller than the subtrahend), pupils should:

• learn to lay out columnar calculations with like digits correctly aligned

• learn to work from right to left, adding or subtracting the least significant digits first Teachers should initially use place-value equipment, such as Dienes, to model the algorithms and help pupils make connections to what they already know about addition and subtraction.


representation

Pupils should use unitising language to describe within-column calculations.



Pupils must also learn to carry out columnar addition calculations that involve regrouping, and columnar subtraction calculations that involve exchange. Regrouping and exchange build on pupils' understanding that 10 ones is equivalent to 1 ten, and that 10 tens is equivalent to 1 hundred. Dienes can be used to model the calculations, and to draw attention to the regrouping/exchange.

Dienes (or any other place-value apparatus) should be used, only initially, to support pupils understanding of the structure of the algorithms, and should not be used as a tool for finding the answer. Once pupils understand the algorithms, they should use known facts to perform the calculation in each column (3NF-1). For calculations with more than 2 addends, pupils should add the digits within a column in the most efficient order.



Figure 22: making efficient choices for within-column calculations when adding 3 addends

Pupils must learn that, although columnar methods can be used for any additive calculation, they are not always the most appropriate choice. For example, 164 + 36 can be calculated by recognising that 64 and 36 is a complement to 100, while 120 + 130 maybe be calculated by unitising in tens (12 tens + 13 tens = 25 tens) or by recognising that 20 + 30 = 50.

Throughout, pupils should continue to recognise the inverse relationship between addition and subtraction. Pupils may represent calculations using partitioning diagrams or bar models, and should learn to check their answers using the inverse operation.



Figure 23: using addition to check a subtraction calculation

You can find out more about fluency for these calculations here in the calculation and fluency section: <u>3AS-2</u>

Making connections

The within-column calculations in columnar addition and subtraction use the facts practised to fluency in <u>3NF-1</u>. Any additive calculation can be carried out using columnar methods. However other methods may sometimes be more efficient, such as those taught in <u>3NPV-3</u> and <u>3NF-3</u>.

3AS-2 Example assessment questions

- 1. Solve these calculations using columnar addition or columnar subtraction.
 - a. 89-23c. 402+130+78e. 345-72b. 127+43+49d. 462-256f. 407-129
- 2. Year 3 want to buy some sports equipment which costs £472. So far they have raised £158. How much more money do they need to raise?
- 3. Cheryl has £135. She spends £53 on some new trainers. How much money does she have left?
- 4. There are 172 non-fiction books in the school library and 356 fiction books. How many books are there in the library altogether?
- 5. Fill in the missing numbers.



6. Mahsa carries out the following columnar addition calculation.

$$\begin{array}{r}
6 & 2 & 8 \\
+ & 1 & 5 & 9 \\
\hline
& 7 & 8 & 7 \\
\hline
& 1 \\
\end{array}$$

Write a columnar subtraction calculation that she could do to check that her calculation is correct.

7. Complete the following calculations. Choose carefully which method to use.



3AS-3 Manipulate the additive relationship

Manipulate the additive relationship:

- Understand the inverse relationship between addition and subtraction, and how both relate to the part–part–whole structure.
- Understand and use the commutative property of addition, and understand the related property for subtraction.

3AS-3 Teaching guidance

Pupils will begin year 3 with an understanding of some of the individual concepts covered in this criterion, and will already be familiar with using partitioning diagrams and bar models. However, pupils need to leave year 3 with a coherent understanding of the additive relationship, and how addition and subtraction equations relate to the various additive structures.

Pupils must understand that the simplest addition and subtraction equations describe the relationship between 3 numbers, where one is a sum of the other two. They should understand that both addition and subtraction equations can be used to describe the same additive relationship. They should practise writing the full set of 8 equations that are represented by a given partitioning diagram or bar model.



25 + 12 = 37	37 - 12 = 25
12 + 25 = 37	37 – 25 = 12
37 = 25 + 12	25 = 37 - 12
37 = 12 + 25	12 = 37 - 25

Figure 24: partitioning diagrams showing the additive relationship between 25, 12 and 37

Pupils should learn and use the correct names for the terms in addition and subtraction equations.

addend + addend = sum minuend – subtrahend = difference

Pupils understanding should go beyond the fact that addition and subtraction are inverse operations. They need understand how the terms in addition and subtraction equations are related to each other, and to the parts and whole within an additive relationship, and use this understanding to manipulate equations.



Figure 25: connecting the terms in addition and subtraction equations to the part-part-whole structure

With experience of the commutative property of addition, pupils can now learn that, because of the relationship between addition and subtraction, the commutative property has a related property for subtraction.

Language focus

"If we swap the values of the subtrahend and difference, the minuend remains the same."

Both of the following equations are therefore correct: 37 - 25 = 1237 - 12 = 25

Pupils should use their understanding of the additive relationship and how it is related to parts and a whole, the inverse relationship between addition and subtraction, and the commutative property, to manipulate equations. They must recognise that if 2 of the 3 numbers in a given additive relationship are known, the unknown number can always be determined: addition is used to find an unknown whole, while subtraction is used to find an unknown part, irrespective of how the problem is presented. For example, 34 + ? = 56 has an unknown part so is solved using subtraction, even though the problem is written as addition. Pupils need to be able to solve:

- missing-addend problems (addend + ? = sum)
- missing-subtrahend problems (minuend -? = difference)
- missing-minuend problems (?- subtrahend = difference)

Pupils have been solving missing-number problems since year 1. However, with smaller numbers, they were able to rely on missing-number facts (known number bonds) or counting on (for example, counting on 3 fingers to get from 12 to 15 for 12 + ? = 15). Now that pupils are using numbers with up to three digits, they will need to rearrange missing-number equations to solve them, using formal written methods where necessary. Teachers should not assume that pupils will be able to do this automatically: pupils will need to spend time learning the properties of the additive relationship, and practising rearranging equations. They should be able to identify the type of each problem in terms

of whether a part or the whole is unknown, and can sketch partitioning diagrams or bar models to help them do this.

Missing-addend problems	Type of problem: missing part Rewrite the addition equation as a subtraction equation, for example: $329 + \boxed{} = 743 \rightarrow 743 - 329 = \boxed{}$ Language focus			
	"There is a missing part. To find the missing part, we subtract the other part from the whole."			
Missing-subtrahend	Type of problem: missing part			
problems	Rewrite the subtraction equation by swapping the subtrahend and the difference, for example: $477 - = 285 \rightarrow 477 - 285 = $			
	Language focus "There is a missing part. To find the missing part, we subtract the other part from the whole."			
Missing-minuend	Type of problem: missing whole			
problems	Rewrite the subtraction equation as an addition equation, for example:			
	$\boxed{}-527=87 \rightarrow 527+87=\boxed{}$			
	Language focus "There is a missing whole. To find the missing whole, we add the 2 parts."			

Making connections

Once pupils have identified the calculation required to solve a missing-number problem, they need to be able to perform that calculation. Pupils must be fluent in identifying and applying appropriate addition and subtraction strategies.

3AS-3 Example assessment questions

Fill in the missing numbers.



3MD-1 Multiplication and division structures

Apply known multiplication and division facts to solve contextual problems with different structures, including quotitive and partitive division.

3MD-1 Teaching guidance

At this stage, pupils will be developing fluency in the 5, 10, 2, 4 and 8 multiplication tables (3NF-2), so should be able to solve multiplication problems about groups of 5, 10, 2, 4 or 8. Pupils have already begun to learn that if the factors are swapped, the product remains the same (3NF-2).

Language focus

"factor times factor is equal to product"

"The order of the factors does not affect the product."

Pupils should also learn that the commutative property allows them to use their known facts to solve problems about 5, 10, 2, 4 or 8 equal groups (for example, 2 groups of 7). An array can be used to illustrate how the commutative property relates to different grouping interpretations – the example below shows that 7 groups of 2 and 2 groups of 7 both correspond to the same total quantity (14).



Figure 26: using an array to show that 7 groups of 2 and 2 groups of 7 both correspond to the same total quantity

This means that pupils can use their knowledge that $7 \times 2 = 14$ to solve a problem about 2 groups of 7, even though they have not yet learned the 7 multiplication table.

Pupils should already be solving division calculations using known division facts corresponding to the 5, 10, 2, 4 and 8 multiplication tables (**3NF–2**). They must also be able to use these known facts to solve both quotitive (grouping) and partitive (sharing) contextual division problems. The same array that was used to illustrate the commutative

property of multiplication can also be used to show how known division facts can be applied to the two different division structures.



At this stage, pupils only need to be able to apply division facts corresponding to division by 5, 10, 2, 4 and 8 to solve division problems with the two different contexts. In year 4, pupils will learn, for example, that if they know that $2 \times 7 = 14$, then they know both $14 \div 2 = 7$ and $14 \div 7 = 2$.

When pupils are solving contextual problems, dividing into groups of 5, 10, 2, 4 or 8 (quotitive division) or sharing into 5, 10, 2, 4 or 8 parts (partitive division), they should calculate by recalling a known multiplication fact rather than by skip counting, as described in **3NF–2** and illustrated above.

Making connections

Using the commutative law of multiplication reduces the number of multiplication tables facts that pupils need to memorise, both in year 3 (3NF-2) and beyond. Being able to calculate the size of a part in partitive contexts using known division facts is a prerequisite for finding unit fractions of quantities (3F-2).

3MD-1 Example assessment questions

1. Circle the expressions that match the picture.

2×6	6×6	6×2
2+6	6+2	6+6

- 2. If one sweet costs 3p, how much do 8 sweets cost?
- 3. I need to buy 32 metres of fencing to go around my garden. The fencing is sold in 8metre lengths. How many 8-metre lengths do I need to buy?
- 4. There are 24 strawberries in a tub. I share them equally between the 4 people in my family. How many does each person get?
- 5. A gardener has 5 plant pots. She plants 6 seeds in each pot. How many seeds does she plant altogether?

3F-1 Use and understand fraction notation

Interpret and write proper fractions to represent 1 or several parts of a whole that is divided into equal parts.

3F–1 Teaching guidance

Pupils should learn that when a whole is divided into equal parts, fraction notation can be used to describe the size of each equal part relative to the whole. Because it is the size of a part relative to the whole which determines the value of a fraction, it is important that pupils talk about, and identify, both the whole and the part from the start of their work on fractions. They should not begin, for example, by talking about '1 out of 3 parts' without reference to a whole.

Pupils should begin by working with concrete resources and diagrams. First they should learn to identify the whole and the number of equal parts, then to describe one particular equal part relative to the whole.



Figure 29: a circle divided into 3 equal parts, with one part shaded

Language focus

"The whole is divided into 3 equal parts. 1 of these parts is shaded."

Pupils must be able to use this precise language to describe a unit fraction of a:

- shape/area (as in the above example)
- measure (for example, a length of ribbon or a beaker of water)
- set (for example, a group of sheep where all are white except one, which is black)

Pupils should then learn to interpret and write unit fractions, relating to these contexts, using mathematical notation. They should continue to describe the whole, the number of parts and the particular part, and relate this to the written fraction.

Say	Write
"The whole has been divided"	The fraction bar: –
"into 3 equal parts."	The denominator: 3
"1 of these parts is shaded."	The numerator: 1

Language focus

"The whole is divided into 3 equal parts. Each part is one-third of the whole."

A clear understanding of unit fractions is the foundation for all future fractions concepts. Pupils should spend sufficient time working with unit fractions to achieve mastery before moving on to non-unit fractions.

Pupils should learn that a non-unit fraction is made up of a quantity of unit fractions. They should practise using unitising language to describe, for example, 5 eighths as 5 one-eighths (here, we are unitising in eighths).

Language focus

"The whole is divided into 8 equal parts and 5 of those parts are shaded.

 $\frac{5}{8}$ of the shape is shaded. $\frac{5}{8}$ is 5 one-eighths."

Pupils should also experience examples where all parts of the shape are shaded (or all parts of the measure or set are highlighted) and the numerator is equal to the denominator. They should understand, for example that $\frac{5}{5}$ represents all 5 equal parts, and is equivalent to the whole.

Teaching should draw attention to the fact that in order to identify a fraction, the parts need to be equal. Comparing situations where the parts are equal and those where they are not is a useful activity (see 3F-1, questions 2 and 4).

Making connections

Showing, describing and representing a unit fraction of a shape, measure or set involves dividing it into a number of equal parts. The theme of dividing a quantity into a given number of equal parts runs through many topics, including:

- partitive division (<u>3MD-1</u>)
- finding a unit fraction of a value using known division facts (<u>3F-2</u>).

3F–1 Example assessment questions

1. What fraction of each diagram is shaded?



2. Does each diagram show the given fraction? Explain your answers.

















3. What fraction of each diagram is shaded/highlighted?



4. Tick or cross each diagram to show whether $\frac{3}{5}$ is shaded. Explain your answers.



5. a. Shade $\frac{1}{10}$ of this set.



b. Shade $\frac{3}{4}$ of this shape.



c. Circle $\frac{4}{5}$ of the flowers.



d. Colour $\frac{1}{3}$ of the line.



3F-2 Find unit fractions of quantities

Find unit fractions of quantities using known division facts (multiplication tables fluency).

3F-2 Teaching guidance

In <u>3F–1</u> pupils learnt how fractions act as operators on a whole. Now they must learn to evaluate the outcome of that operation, for unit fractions of quantities, and connect this to what they already know about dividing quantities into equal parts using known division facts, for example:

$30\div10=3$	$36 \div 4 = 9$
SO	SO
$\frac{1}{10}$ of 30 = 3	$\frac{1}{4}$ of $36 = 9$

In year 3, pupils learn the 5, 10, 2, 4 and 8 multiplication tables, so examples in this criterion should be restricted to finding $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{2}$, $\frac{1}{4}$ or $\frac{1}{8}$ of quantities. This allows pupils to focus on the underlying concepts instead of on calculation.

Pupils should begin by working with concrete resources or diagrams representing sets. As in **3F–1**, they should identify the whole, the number of equal parts, and the size of each part relative to the whole written as a unit fraction. They should then extend their description to quantify the number of items in a part, and connect this to the unit-fraction operator.



Figure 30: 12 oranges divided into 4 equal parts



Once pupils can confidently and accurately describe situations where the value of a part is visible, they should learn to calculate the value of a part when it cannot be seen. Bar models are a useful representation here. Pupils should calculate the size of the parts using known division facts, initially with no reference to unit fractions. This is partitive division (<u>3MD-1</u>).





Figure 31: using a bar model to represent 15 divided into 5 equal parts

Then pupils should make the link between partitive division and finding a fraction of a quantity. They should understand that the situations are the same because both involve dividing a whole into a given number of equal parts. Pupils must understand that, therefore, division facts can be used to find a unit fraction of a quantity.







Figure 32: using a bar model to represent one-fifth of 15

Language focus

"To find $\frac{1}{5}$ of 15, we divide 15 into 5 equal parts."

"15 divided by 5 is equal to 3, so $\frac{1}{5}$ of 15 is equal to 3."

Making connections

This criterion builds directly on 3F-1, where pupils learnt to associate fraction notation with dividing a shape, measure or set into a number of equal parts.

The focus of this criterion is understanding that finding a unit fraction of a quantity is the same structure as partitive division (3MD-1).

In <u>3NF-1</u> pupils develop fluency in the 5, 10, 2, 4 and 8 multiplication tables and associated division facts. These division facts are applied here to find $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{2}$, $\frac{1}{4}$ or $\frac{1}{8}$ of quantities.

3F-2 Example assessment questions

- 1. Rohan saved £32. He spends $\frac{1}{4}$ of his money on a toy. How much does he spend?
- 2. Find:
 - a. $\frac{1}{5}$ of 35
 - b. $\frac{1}{10}$ of 40
 - c. $\frac{1}{8}$ of 24
- 3. The school caretaker buys 50 litres of paint. She uses $\frac{1}{5}$ of it to paint the year 3 classroom. How many litres of paint is this?
- 4. There are 16 apples in a fruit bowl. Some children eat $\frac{1}{4}$ of the apples. How many are left?

3F–3 Fractions within 1 in the linear number system

Reason about the location of any fraction within 1 in the linear number system.

3F-3 Teaching guidance

So far (in <u>3F-1</u> and <u>3F-2</u>) pupils will probably have only experienced fractions as operators, for example, $\frac{1}{3}$ of this shape, $\frac{3}{5}$ of this line, or $\frac{1}{4}$ of this quantity. Pupils must also develop an understanding of fractions as numbers, each of which has a place in the linear number system (for example, the number $\frac{1}{4}$ in contrast to the operator $\frac{1}{4}$ of something). Pupils will already have learnt to place whole numbers on number lines, and now they must learn that other numbers (fractions) lie between these whole numbers, beginning in year 3 with fractions within 1.

Pupils should learn to count, forwards and backwards, in multiples of unit fractions, with the support of number lines.



Figure 33: number line to support counting to 1 in multiples of one quarter



Figure 34: number line to support counting to 1 in multiples of one fifth

Pupils should practise dual counting.

Language focus

"One fifth, two fifths, three fifths..."

"1 one-fifth, 2 one-fifths, 3 one-fifths..."

This reinforces the understanding that non-unit fractions are repeated additions of unit fractions.

Pupils also need to understand that a fraction with the numerator equal to the denominator is equivalent to 1. Practice should involve counting, for example, both "… three-fifths, four-fifths, five-fifths" and "… three-fifths, four-fifths, one".

Pupils should then learn to label marked number lines, within 1. Identifying points between labelled intervals is an important skill in graphing and measures. A common mistake that pupils make is to count the number of marks between labelled intervals, rather than the number of parts, for example, on the number line below they may count 3 marks and incorrectly deduce that the number line is marked in thirds.



Figure 35: labelling a 0 to 1 number line marked in quarters

Language focus

"Each whole-number interval is divided into 4 equal parts, so we count in quarters."

Pupils must also be able to estimate the value or position of fractions on 0 to 1 number lines that do not have fractional marks.



Figure 36: estimating the position of a fraction on a 0 to 1 number line

Pupils also need to be able to reason, for example, that $\frac{1}{4}$ is nearer to 0 than $\frac{1}{3}$ is, because $\frac{1}{4}$ is smaller than $\frac{1}{3}$; they should consider the number of parts the 0 to 1 interval is divided into, and understand that the greater the denominator, the more parts there are, and therefore $\frac{1}{4} < \frac{1}{3}$.



Figure 37: estimating the relative positions of one quarter and one third on a 0 to 1 number line

Making connections

Having a visual image of fractions in the linear number system helps pupils to add and subtract fractions with the same denominator, for example $\frac{5}{6} - \frac{3}{6}$ (3F-4). It also supports comparison of fractions, and reading scales.

3F–3 Example assessment questions

1. Label the points on this number line.



2. How tall is this plant? Give your answer as a fraction of a metre.



- 3. a. Which is larger, $\frac{6}{8}$ or $\frac{3}{8}$? Explain your answer.
 - b. Which is larger, $\frac{1}{4}$ or $\frac{1}{3}$? Explain your answer.

4. Gemma and Kasper look at this number line.



Who is correct? Explain your answer.

5. Add the missing labels to the measuring jug.



3F-4 Add and subtract fractions within 1

Add and subtract fractions with the same denominator, within 1.

3F-4 Teaching guidance

To add and subtract fractions, pupils must already understand that non-unit fractions are repeated additions of unit fractions, for example, three-eighths is 3 one-eighths. In other words, pupils must have begun to unitise with unit fractions in the same way that they learnt to unitise, for example, in tens (30 is 3 tens). Addition and subtraction of fractions with the same denominator then follows logically: just as pupils learnt that 3 tens plus 2 tens is 5 tens, they can reason that 3 one-eighths plus 2 one-eighths is equal to 5 one-eighths.





Figure 38: adding and subtracting fractions with the same denominator: pie model, bar model and number line



Pupils should be able to understand and use the following generalisations.

Language focus

"When adding fractions with the same denominators, just add the numerators."

"When subtracting fractions with the same denominators, just subtract the numerators."

Making connections

In <u>3F-1</u> and <u>3F-3</u>, pupils learnt that non-unit fractions are made up of multiples of a unit fraction. This criterion is based on pupils being able to unitise with unit fractions. Pupils already learnt to unitise in tens and combine this understanding with known addition and subtraction facts in <u>3NPV-1</u> and <u>3NF-3</u>.

3F-4 Example assessment questions

1. Complete the calculations.



2. Diego writes:

 $\frac{3}{12} + \frac{5}{12} = \frac{8}{12}$

Mark writes:

 $\frac{3}{12} + \frac{5}{12} = \frac{8}{24}$

Who is correct? Explain the mistake that has been made.

3. Decide whether each calculation is correct or not. Explain your answers.

	Correct (✓) or incorrect (×)?	Explanation
$\frac{7}{12} - \frac{2}{12} = \frac{5}{12}$		
$\frac{4}{7} - \frac{2}{7} = \frac{2}{0}$		
$\frac{8}{10} - \frac{2}{10} - \frac{1}{10} = \frac{3}{10}$		
$\frac{7}{9} - \frac{7}{9} = 0$		
$\frac{5}{8} - \frac{2}{8} - \frac{2}{8} = \frac{1}{8}$		

4. Sofia had a jug containing $\frac{7}{10}$ of a litre of juice. She drank $\frac{4}{10}$ of a litre. How much does she have left?



3G–1 Recognise right angles

Recognise right angles as a property of shape or a description of a turn, and identify right angles in 2D shapes presented in different orientations.

3G-1 Teaching guidance

Pupils must have learnt about fractions before beginning work on this criterion. In particular they should recognise one-, two- and three-quarters of a circle.

Pupils must be able to describe and represent quarter, half and three-quarter turns (clockwise and anti-clockwise). Pupils should begin by making quarter turns with their bodies, following instructions such as "Stand, and make a quarter turn clockwise. Walk in a straight line. Stop. Make a quarter turn anticlockwise." They should be able to relate these movements to the quarter turn of a clock hand. Pupils should learn that the angle relative to the starting orientation, created by a quarter turn (in either direction), is called a right angle – programmable robots and geo-strips are useful tools for illustrating this. Pupils should then learn to follow instructions involving $\frac{1}{2}$ turn and $\frac{3}{4}$ turns, clockwise is the same as the result of making a $\frac{1}{2}$ turn anticlockwise. Pupils should also understand $\frac{1}{2}$ and $\frac{3}{4}$ turns as repeated $\frac{1}{4}$ turns, and therefore as repeated turns through a right angle.

Pupils should recognise that a right angle is the 'amount of turn' between 2 lines, and is independent of the length of those lines. They should be presented with a right angle created by 2 long lines (such as two metre sticks) and a right angle created by 2 short lines (such as 2 geo-strips), and understand that both are right angles. It is important that pupils know that it is incorrect to describe the right-angle made from longer lines as a 'bigger right angle', or that made from the shorter lines as a 'smaller right angle'.

Pupils should practise identifying right angles in their environment, for example, the corner of their desk, the panes of a window, or the hands on a clock at 3pm or 9pm. They should learn to use various tools to confirm that angles are right angles, for example a card circle with a quarter circle cut out, or a piece of paper of any size that has been folded in half and half again to create a right angle.

Pupils must then learn to identify right angles in polygons. This should involve both handling shapes (for example, cut from cardboard) and working with images of shapes. When pupils are handling shapes, they should practise rotating the shapes to check each angle against a right-angle checker. Images of shapes should be presented in a variety of orientations, so that pupils' ability to identify right angles is not dependent on the lines being horizontal and vertical. Pupils must also learn to interpret and use the standard convention for marking right angles (as illustrated below).



Figure 39: irregular pentagon with 3 right angles marked using standard convention

Whether working with cut-out shapes or images, pupils should be able to state whether a given angle is greater than or smaller than a right angle, using the angle-checker.

Pupils should recognise that:

- the only polygon in which every vertex can be a right angle is a quadrilateral
- quadrilaterals that have 4 right angles are rectangles irrespective of the length of their sides
- a quadrilateral that has all side-lengths equal and every vertex a right angle is a regular rectangle that can also be called a square

Making connections

In <u>3G-2</u>, children will learn that 2 lines are at right angles are termed 'perpendicular'. Composing and drawing shapes in **3G-2** provides another context in which to identify right angles.

3G–1 Example assessment questions

1. Here is a map of a treasure island.



- a. Follow the instructions and say where you end up. Each time, start at the camp, facing north.
 - i. Go forwards 3 squares. Make a quarter turn clockwise. Go forwards 2 squares. Make a quarter turn anticlockwise. Go forwards 2 squares. Where are you?
 - ii. Make a three-quarter turn clockwise.Go forward 3 squares.Make a quarter turn anticlockwise.Go forward 1 square.Where are you?
- b. Start at the camp, facing North. Write some instructions, like the ones above, to get to the treasure.
- 2. Draw an irregular hexagon with one right angle on this grid.

•	•	•	•	•	•	•
•	•	•	•	•	•	•
•	•	•	•	•	•	•
•	•	•	•	•	•	•
•	•	•	•	•	•	•
•	•	•	•	•	•	•
•			•	•		•

3. Mark all of the right angles in these shapes. Use a right-angle checker to help you.



3G–2 Draw polygons and identify parallel and perpendicular sides

Draw polygons by joining marked points, and identify parallel and perpendicular sides.

3G–2 Teaching guidance

Pupils must learn to draw polygons by joining marked points, precisely, using a ruler. Pupils should be able to mark vertices themselves on a grid (square or isometric), as well as join already-marked points.

Pupils must be able to identify a pair of parallel or perpendicular lines, as well as horizontal and vertical lines. They should be able to explain why a pair of lines are parallel or perpendicular.

Language focus

"These 2 lines are parallel because they are always the same distance apart. They will never meet no matter how far we extend them."

"These 2 lines are perpendicular because they are at right angles to each other."

Pupils should be able to select or create shapes according to parameters that include these terms, such as joining 2 isosceles triangles to make a parallelogram.

Pupils may use standard notation to mark parallel sides.

Making connections

In <u>3G–1</u>, children learnt to identify right angles. In this criterion they should identify right angles in shapes they have drawn or made, and know that a right angle is made at the point where two perpendicular lines meet.

3G–2 Example assessment questions

- 1. Task: Provide each pupil with 2 trapezium pieces from a pattern block set. Then ask them to make 3 different shapes by joining the pieces and discuss the properties of each shape they make.
- 2. Here are 5 vertices of a regular hexagon. Mark the sixth vertex and join the points to draw the hexagon.



3. Here are 2 sides of a square. Complete the square.



4. Look at these 5 quadrilaterals. Mark all the pairs of parallel sides. Hint: you can extend sides to help you.



5. Mark the missing vertex of this quadrilateral so that 2 of the sides are perpendicular.



Calculation and fluency

Number, place value and number facts: 3NPV-2 and 3NF-3

- **3NPV-2:** Recognise the place value of each digit in *three*-digit numbers, and compose and decompose *three*-digit numbers using standard and non-standard partitioning.
- **3NF–3:** Apply place-value knowledge to known additive and multiplicative number facts (scaling facts by 10), for example:

8 + 6 = 14 and $14 - 6 = 8$	$3 \times 4 = 12$ and $12 \div 4 = 3$
SO	SO
80 + 60 = 140 and $140 - 60 = 80$	$30 \times 4 = 120$ and $120 \div 4 = 30$

Representations such as place-value counters and partitioning diagrams (<u>3NPV-2</u>), and tens-frames with place-value counters (<u>3F-3</u>), can be used initially to help pupils understand calculation strategies and make connections between known facts and related calculations. However, pupils should not rely on such representations for calculating. For the calculations in **3NF-3**, for example, pupils should instead be able to calculate by verbalising the relationship.

Language focus

"8 plus 6 is equal to 14, so 8 tens plus 6 tens is equal to 14 tens."

"14 tens is equal to 140."

Pupils should be developing fluency in both formal written and mental methods for addition and subtraction. Mental methods can include jottings to keep track of calculation, or language structures as exemplified above. Pupils should select the most efficient method to calculate depending on the numbers involved.

3NF-1 Fluently add and subtract within and across **10**

Secure fluency in addition and subtraction facts that bridge 10, through continued practice.

Pupils who are fluent in addition and subtraction facts within and across 10 have the best chance of mastering columnar addition and columnar subtraction. Teachers should make sure that fluency in addition and subtraction facts is given the same prominence as fluency in multiplication tables.

Pupils should continue to practise calculating with additive facts within 10.

Pupils may initially use manipulatives, such as tens frames and counters, to apply the strategies for adding and subtracting across 10 described in year 2 (**2AS-1**). However, they should not be using the manipulatives as a tool for finding answers, and by the end of year 3 pupils should be able to carry out these calculations mentally, using their fluency in complements to 10 and partitioning.

Pupils do not need to memorise all additive facts for adding and subtracting across 10, but need to be able to recall appropriate doubles (double 6, 7, 8 and 9) and corresponding halves (half of 12, 14, 16 and 18), and use these known facts for calculations such as 6+6=12 and 18-9=9.

3AS-2 Columnar addition and subtraction

Add and subtract up to three-digit numbers using columnar methods.

Pupils must be able to add 2 or more numbers using columnar addition, including calculations whose addends have different numbers of digits.

	2	7	4			6	2		1	8	6
+	3	5	4	+	• 4	8	1			5	7
	6	2	8		5	4	3	+	4	3	4
	1				1			-	6	7	7
								-	1	1	

Figure 40: columnar addition for calculations involving three-digit numbers

For calculations with more than 2 addends, pupils should add the digits within a column in the most efficient order. For the third example above, efficient choices could include:

- beginning by making 10 in the ones column
- making double 8 in the tens column

Pupils must be able to subtract 1 three-digit number from another using columnar subtraction. They should be able to apply the columnar method to calculations where the subtrahend has fewer digits than the minuend, and they must be able to exchange through 0.

	51 Ø2	8	$\overset{4}{\cancel{5}}\overset{1}{\cancel{5}}\overset{1}{\cancel{5}}$	$\overset{2}{\mathcal{Z}}$	0 19	1 2
-	2 7	4	- 78 -	1	5	4
	35	4	4 7 8	1	4	8

Figure 41: columnar subtraction for calculations involving three-digit numbers

Pupils should make sensible decisions about how and when to use columnar subtraction. For example, when the minuend and subtrahend are very close together pupils may mentally find the difference, avoiding the need for column subtraction. For example, for 402 - 398, pupils can see that 398 is 2 away from 400, and then there is 2 more to get to 402, so the difference is 4. This is more efficient than the corresponding columnar subtraction calculation which requires exchange through the zero.

3NF-2 Recall of multiplication tables

Recall multiplication facts, and corresponding division facts, in the 10, 5, 2, 4 and 8 multiplication tables, and recognise products in these multiplication tables as multiples of the corresponding number.

Pupils who are fluent in these multiplication table facts can solve the following types of problem by automatic recall of the relevant fact rather than by skip counting or reciting the relevant multiplication table:

- identifying products $8 \times 4 =$ $10 \times 10 =$
- solving missing-factor problems

$3\times 5 = 45$	6× = 48	22 = 🔤 × 2
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• using relevant multiplication table facts to solve division problems

35 ÷ 5 =		$=40\div8$
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Pupils should also be fluent in interpreting contextual multiplication and division problems, identifying the appropriate calculation and solving it using automatic recall of the relevant fact. This is discussed, and example questions are given, in <u>3MD–1</u>.

As pupils become fluent with the multiplication table facts, they should also develop fluency in related calculations as described in 3NF-3 (scaling number facts by 10).





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